Large Non-Gaussianity in Axion Inflation

Marco Peloso, University of Minnesota

Neil Barnaby, M.P., PRL 106, 181301 (2011)

Neil Barnaby, Ryo Namba, M.P., JCAP 009, 1104, (2011)

Large Non-Gaussianity in Axion Inflation

Marco Peloso, University of Minnesota

Neil Barnaby, M.P., PRL 106, 181301 (2011)

Neil Barnaby, Ryo Namba, M.P., JCAP 009, 1104, (2011)

- Effect of $\frac{\alpha}{f} \phi \, F \, \tilde{F}$ on primordial perturbations

Large Non-Gaussianity in Axion Inflation

Marco Peloso, University of Minnesota

Neil Barnaby, M.P., PRL 106, 181301 (2011)

Neil Barnaby, Ryo Namba, M.P., JCAP 009, 1104, (2011)

- Effect of $\frac{\alpha}{f} \phi \, F \, \tilde{F}$ on primordial perturbations

- $f \sim 10^{-2} \, \alpha \, M_p \;\; \rightarrow$ detectable non-gaussianity of characteristic ($\sim$ equilateral) shape

- Virtue of inflation: simplest models (single, standard kinetic term, slowly rolling field) work ! Unobservable primordial non-gaussianity.

- Non-gaussianity $\leftrightarrow$ inflaton interactions. ϕ not free: (i) gravity; (ii) reheating; make sure compatible with the flatness of $V(\phi)$

- Virtue of inflation: simplest models (single, standard kinetic term, slowly rolling field) work ! Unobservable primordial non-gaussianity.

- Non-gaussianity $\leftrightarrow$ inflaton interactions. ϕ not free: (i) gravity; (ii) reheating; make sure compatible with the flatness of $V(\phi)$

By itself, flatness stringent requirement:

For example $\Delta V = \frac{\lambda}{4}\phi^4$, $\lambda \simeq 10^{-13}$. Or, for example, even higher dim.

Planck – suppressed operators can spoil inflaton $V = e^{\frac{\phi^2}{M_p^2}}\left(DW^2 - \frac{3W^2}{M_p^2}\right)$

- Virtue of inflation: simplest models (single, standard kinetic term, slowly rolling field) work ! Unobservable primordial non-gaussianity.

- Non-gaussianity $\leftrightarrow$ inflaton interactions. ϕ not free: (i) gravity; (ii) reheating; make sure compatible with the flatness of $V(\phi)$

By itself, flatness stringent requirement:

For example $\Delta V = \frac{\lambda}{4}\phi^4$, $\lambda \simeq 10^{-13}$. Or, for example, even higher dim.

Planck – suppressed operators can spoil inflaton $V = e^{\frac{\phi^2}{M_p^2}}\left(DW^2 - \frac{3W^2}{M_p^2}\right)$

- Here, single field slow roll inflation, for which coupling to "matter" provides large non-gaussianity, while $V(\phi)$ controllably flat.

QCD axion $\rightarrow$ Inflaton axion

QCD instantons $\rightarrow \begin{cases} \Delta\mathcal{L} = \frac{-g^2}{16\pi^2}\,\theta\, F\,\tilde{F} \\ V = \Lambda^4\,[1 - \cos\theta] \end{cases}$

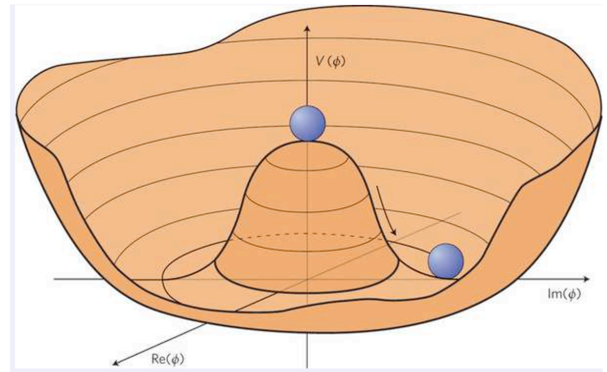
Limit neutron electric dipole moment $\Rightarrow$ $\theta \lesssim 10^{-10}$

QCD axion $\longrightarrow$ Inflaton axion

QCD instantons $\rightarrow \begin{cases} \Delta\mathcal{L} = \frac{-g^2}{16\pi^2}\,\theta\, F\,\tilde{F} \\ V = \Lambda^4\,[1 - \cos\theta] \end{cases}$

Limit neutron electric dipole moment $\Rightarrow$ $\theta \lesssim 10^{-10}$

Peccei, Quinn '77: Chiral U(1) symmetry spontaneously broken $\Phi = (f + \rho)\, e^{i\phi/f}$



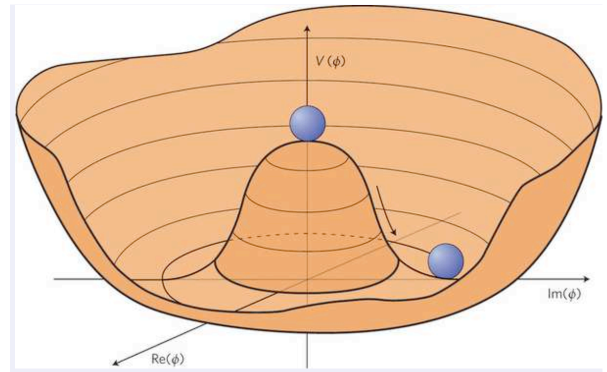
Symmetry is anomalous $\Rightarrow$ $\theta \rightarrow \theta + \frac{\phi}{f}$

QCD axion $\rightarrow$ Inflaton axion

QCD instantons $\rightarrow$ $\begin{cases} \Delta\mathcal{L} = \frac{-g^2}{16\pi^2}\,\theta\,F\,\tilde{F} \\ V = \Lambda^4\,[1 - \cos\theta] \end{cases}$

Limit neutron electric dipole moment $\Rightarrow$ $\theta \lesssim 10^{-10}$

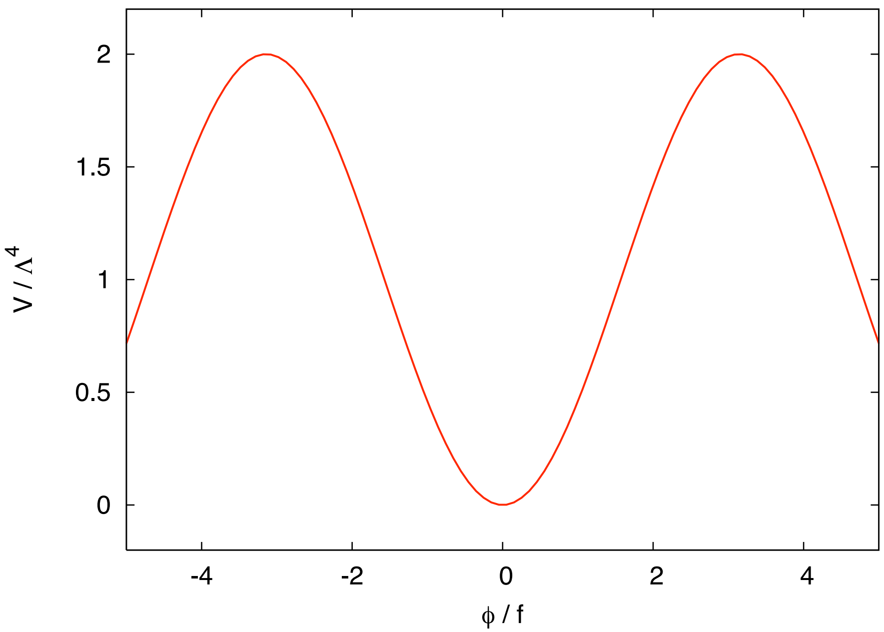
Peccei, Quinn '77: Chiral U(1) symmetry spontaneously broken $\Phi = (f + \rho)\, e^{i\phi/f}$

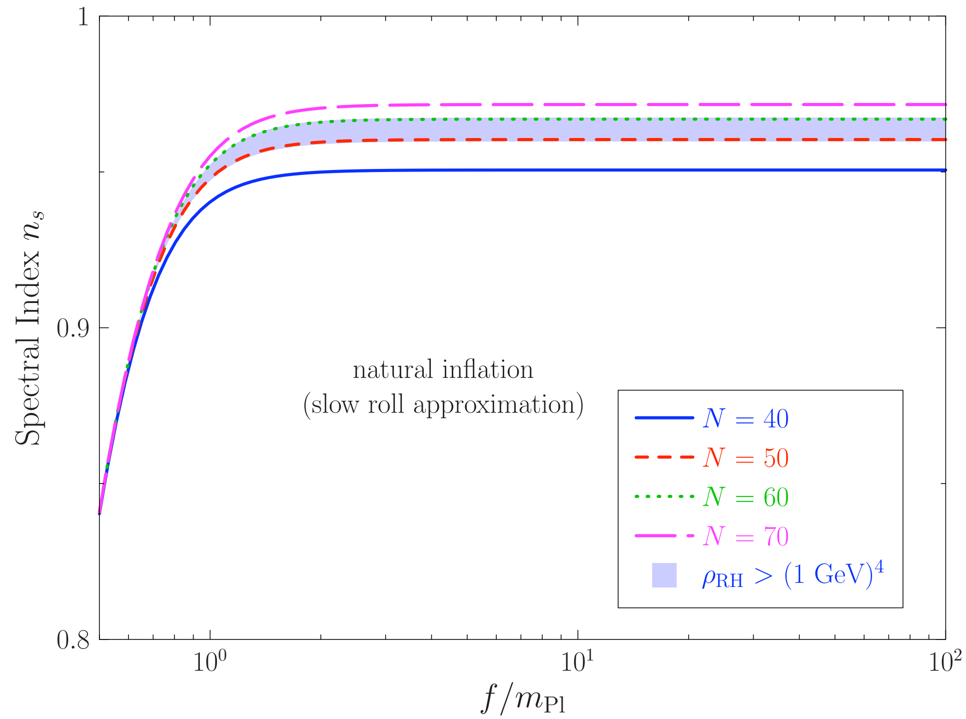


Symmetry is anomalous $\Rightarrow$ $\theta \rightarrow \theta + \frac{\phi}{f}$

- Smallness of Λ is technically natural. No perturbative ~~shift~~
- UV completion (ρ) relevant only at scales $> f$
- ϕ only derivatively coupled

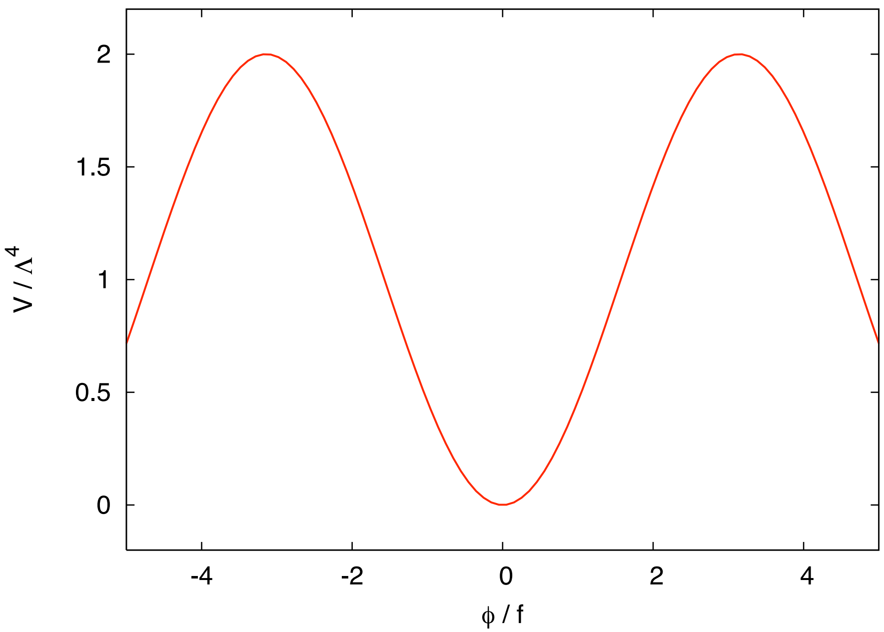
Natural Inflation: Freese, Frieman, Olinto '90

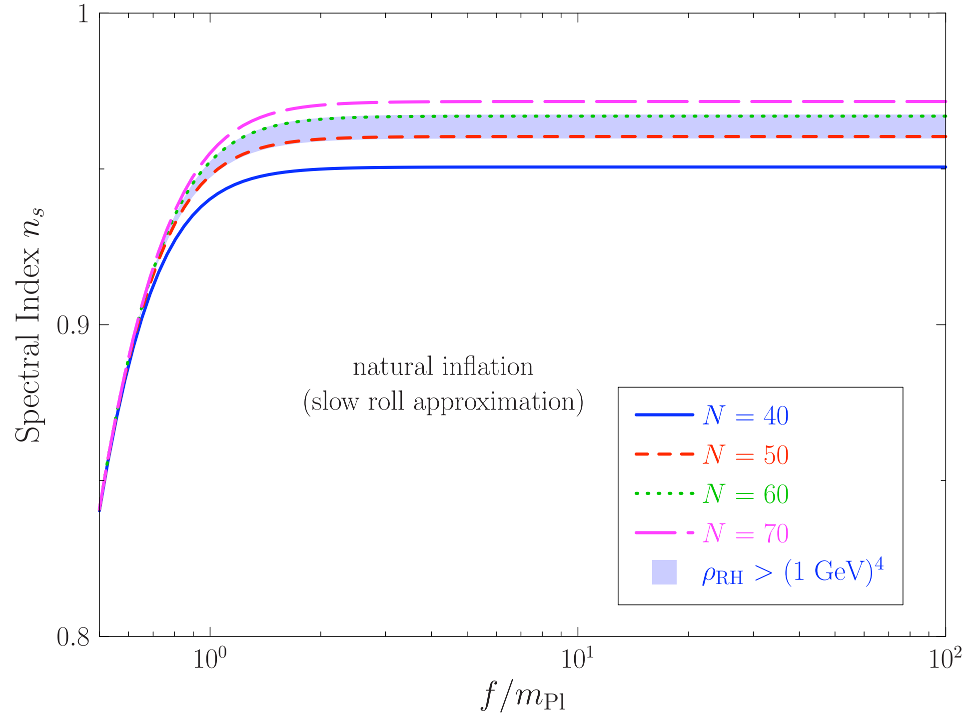




Savage, Freese, Kinney '06

Natural Inflation: Freese, Frieman, Olinto '90





Savage, Freese, Kinney '06

Problems with $f > M_p$

- $U(1)_{\rm PQ}$ broken above QG scale
- Hard in weakly coupled string theory

Kallosh, Linde, Linde, Susskind '95

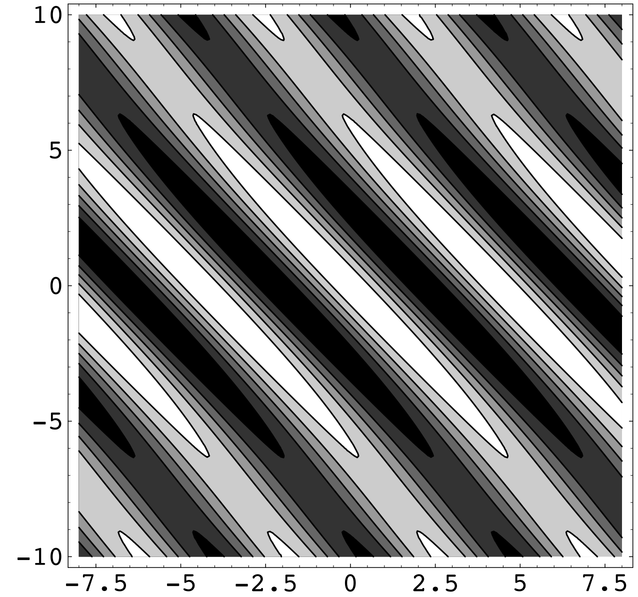
Banks, Dine, Fox, Gorbatov '03

Two axions & gauge groups

Kim, Nilles, MP '04

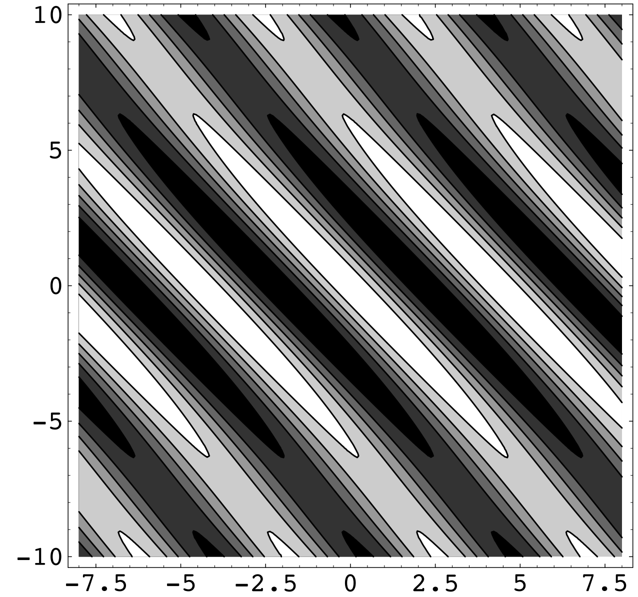
$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

$f_{\text{eff}} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$



Two axions & gauge groups

Kim, Nilles, MP '04



$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

$f_{\text{eff}} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$

N-flation

Dimopoulos et al '05

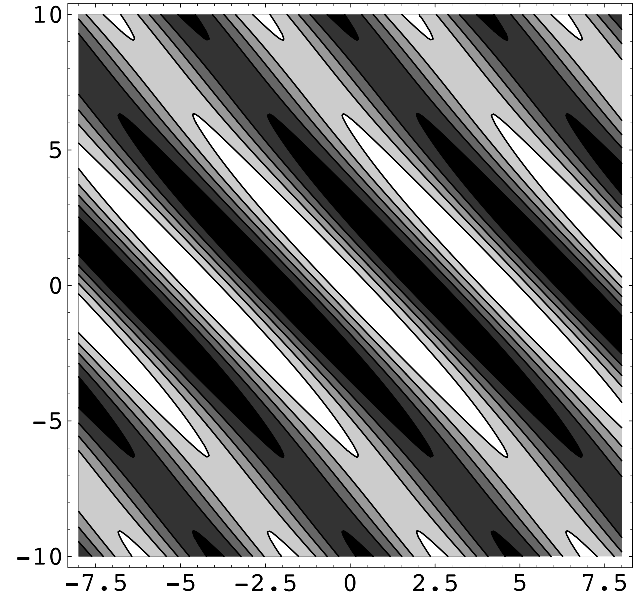
Collectively drive inflation, $f_{\text{eff}} = \sqrt{N}\, f$

Two axions & gauge groups

Kim, Nilles, MP '04

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1} \right) \right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2} \right) \right]$$

$f_{\text{eff}} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$



N-flation

Dimopoulos et al '05

Collectively drive inflation, $f_{\text{eff}} = \sqrt{N}\, f$

Axion monodromy

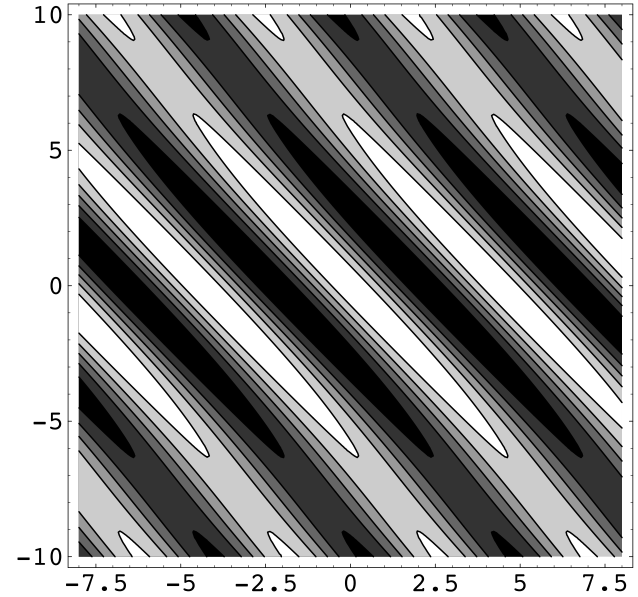
McAllister, Silverstein, Westphal '08

Flauger et al '09

$\Delta V \propto \phi$ from brane wrapping

Two axions & gauge groups

Kim, Nilles, MP '04



$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

$f_{\rm eff} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$

N-flation

Dimopoulos et al '05

Collectively drive inflation, $f_{\rm eff} = \sqrt{N}\, f$

Axion monodromy

McAllister, Silverstein, Westphal '08

Flauger et al '09

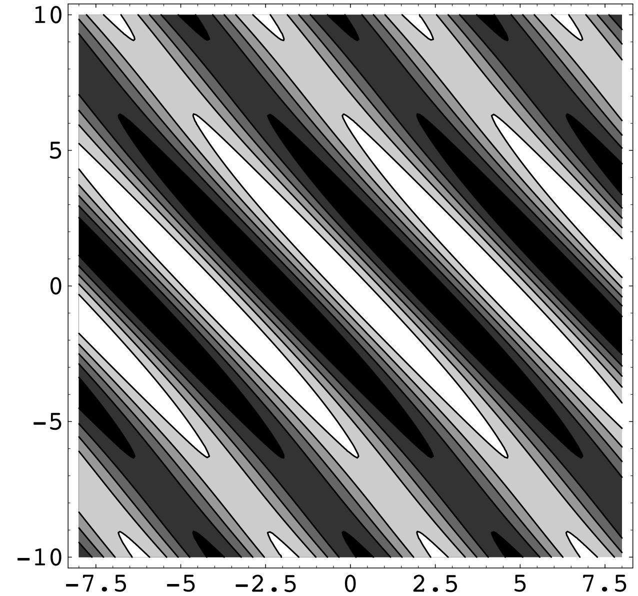
$\Delta V \propto \phi$ from brane wrapping

Dante's inferno

Berg, Pajer, Sjors '09

Two axions & gauge groups

Kim, Nilles, MP '04



$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

$f_{\text{eff}} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$

N-flation

Dimopoulos et al '05

Collectively drive inflation, $f_{\text{eff}} = \sqrt{N}\, f$

Axion monodromy

McAllister, Silverstein, Westphal '08

Flauger et al '09

$\Delta V \propto \phi$ from brane wrapping

Dante's inferno

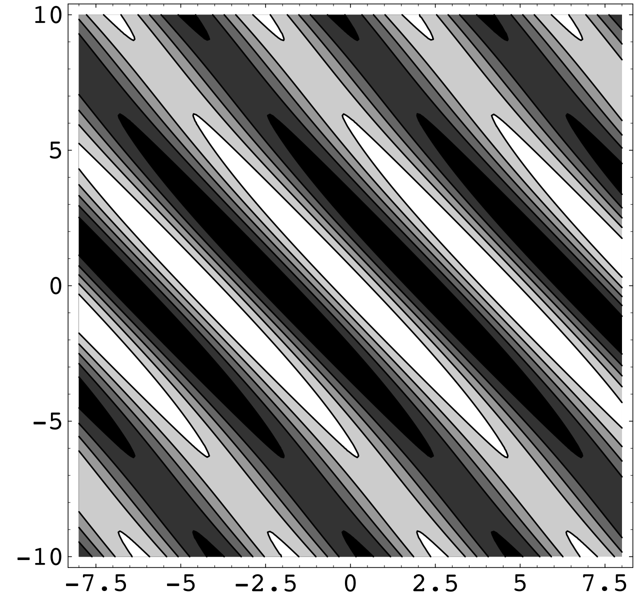
Berg, Pajer, Sjors '09

Axion-4form mixing

Kaloper, Sorbo '08

Two axions & gauge groups

Kim, Nilles, MP '04



$$V = \Lambda_1^4\left[1-\cos\left(\frac{\theta}{f_1}+\frac{\rho}{g_1}\right)\right]+\Lambda_2^4\left[1-\cos\left(\frac{\theta}{f_2}+\frac{\rho}{g_2}\right)\right]$$

$f_{\rm eff} >> f, g$ if $f_1/g_1 \simeq f_2/g_2$

N-flation

Dimopoulos et al '05

Collectively drive inflation, $f_{\rm eff} = \sqrt{N}\, f$

Axion monodromy

McAllister, Silverstein, Westphal '08

Flauger et al '09

$\Delta V \propto \phi$ from brane wrapping

Dante's inferno

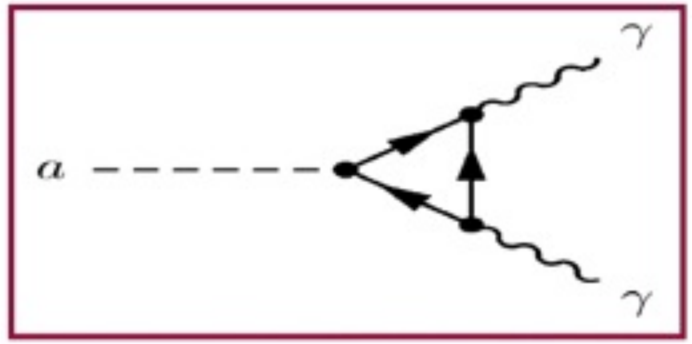
Berg, Pajer, Sjors '09

Axion-4form mixing

Kaloper, Sorbo '08

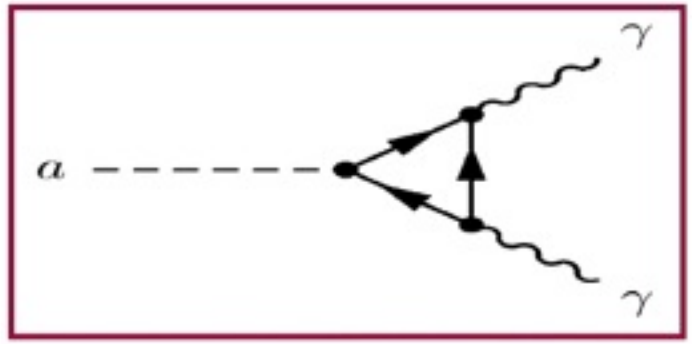
Controllable realizations of large field inflation $(V \propto \phi,\, \phi^2)$, with $f \ll M_p$

$$\mathcal{L} \supset -\frac{\alpha}{f}\phi F \tilde{F}$$



- Dictated by shift-symmetry and parity
- Generally present, not “extra ingredient”

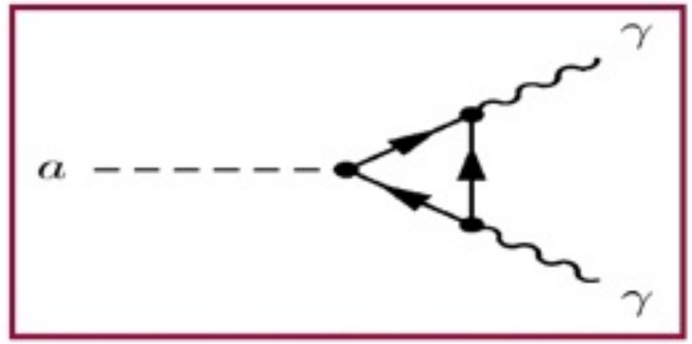
$$\mathcal{L} \supset -\frac{\alpha}{f}\phi F \tilde{F}$$



- Dictated by shift-symmetry and parity
- Generally present, not "extra ingredient"

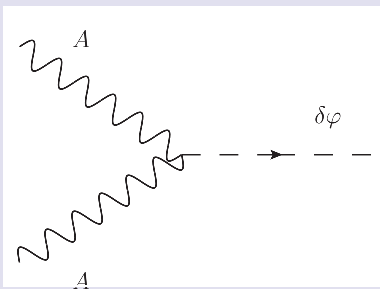
1. $\varphi^{(0)} \rightarrow A + A$, non-perturbative depletion $\propto \dot{\varphi}^{(0)}$
   $\Longrightarrow$ Exponential growth of A

$$\mathcal{L} \supset -\frac{\alpha}{f}\phi F \tilde{F}$$



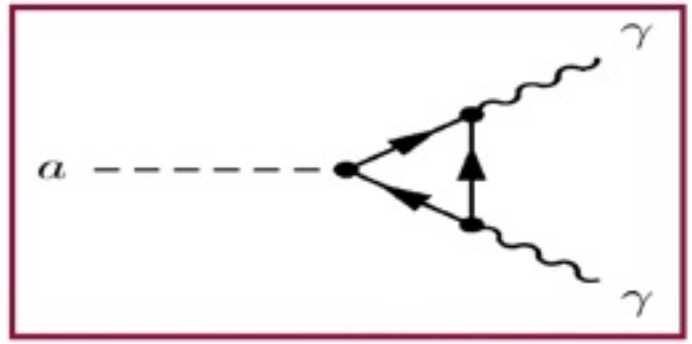
- Dictated by shift-symmetry and parity
- Generally present, not "extra ingredient"

1. $\varphi^{(0)} \to A + A$, non-perturbative depletion $\propto \dot{\varphi}^{(0)}$
   $\Longrightarrow$ Exponential growth of A
2. $A + A \to \delta\varphi$, inverse decay



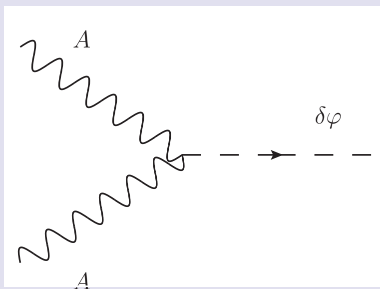
$\Longrightarrow$ Significant contribution to $\delta\varphi$!

$$\mathcal{L} \supset -\frac{\alpha}{f}\,\phi\,F\,\tilde{F}$$



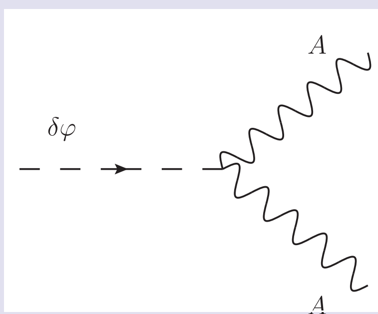
- Dictated by shift-symmetry and parity
- Generally present, not "extra ingredient"

1. $\varphi^{(0)} \to A + A$, non-perturbative depletion $\propto \dot{\varphi}^{(0)}$
   $\Longrightarrow$ Exponential growth of A
2. $A + A \to \delta\varphi$, inverse decay



$\Longrightarrow$ Significant contribution to $\delta\varphi$!

3. $\delta\varphi \to A + A$, perturbative decay



$\Longrightarrow$Important only AFTER inflation (reheating)

$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\phi^{(0)}\ F\,\tilde{F}$$

$$A''_{\pm} + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_{\pm} = 0$$

helicity

$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\,\phi^{(0)}\; F\,\tilde{F}$$

$$A''_{\pm} + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_{\pm} = 0$$

helicity

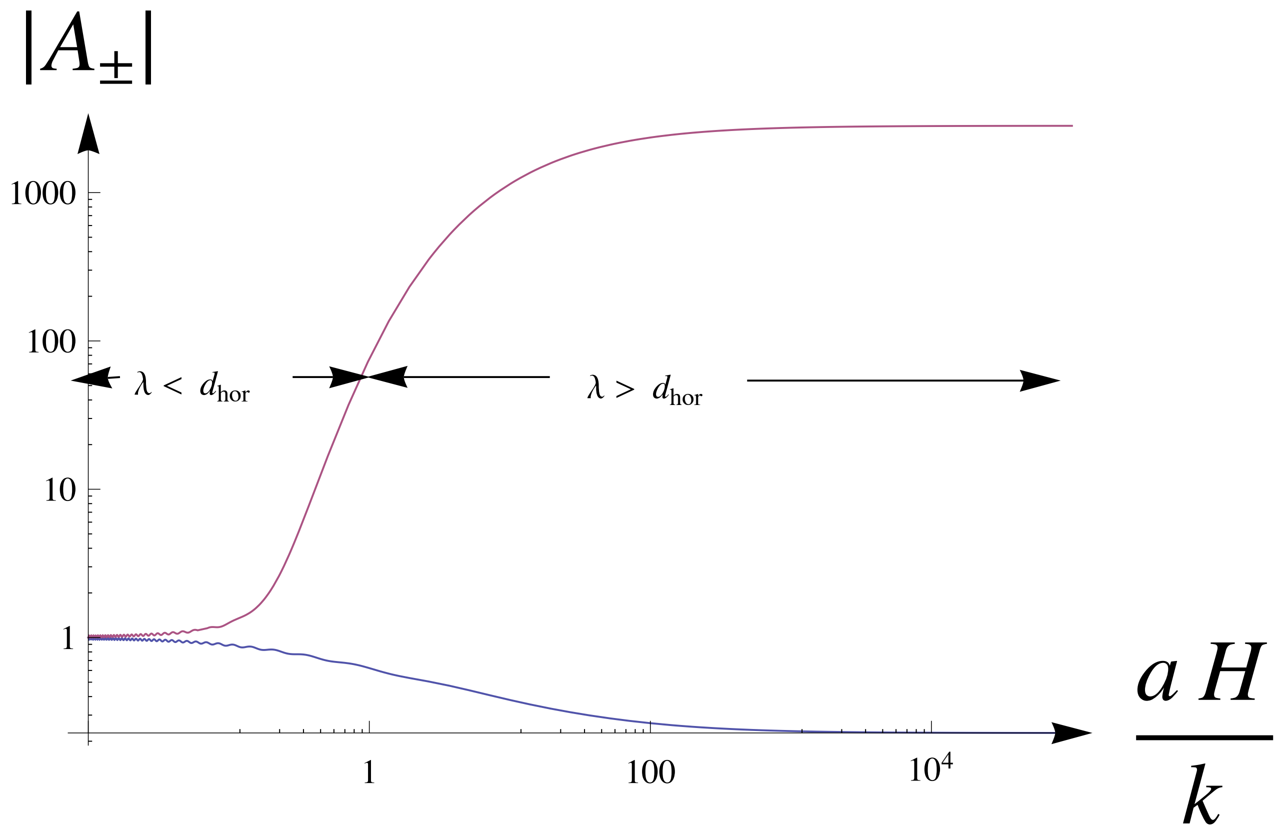
$a\,\dot{\phi}^{(0)}$

$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\phi^{(0)}\ F\tilde{F}$$

$$A''_\pm + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_\pm = 0$$

helicity

$a\,\dot{\phi}^{(0)}$



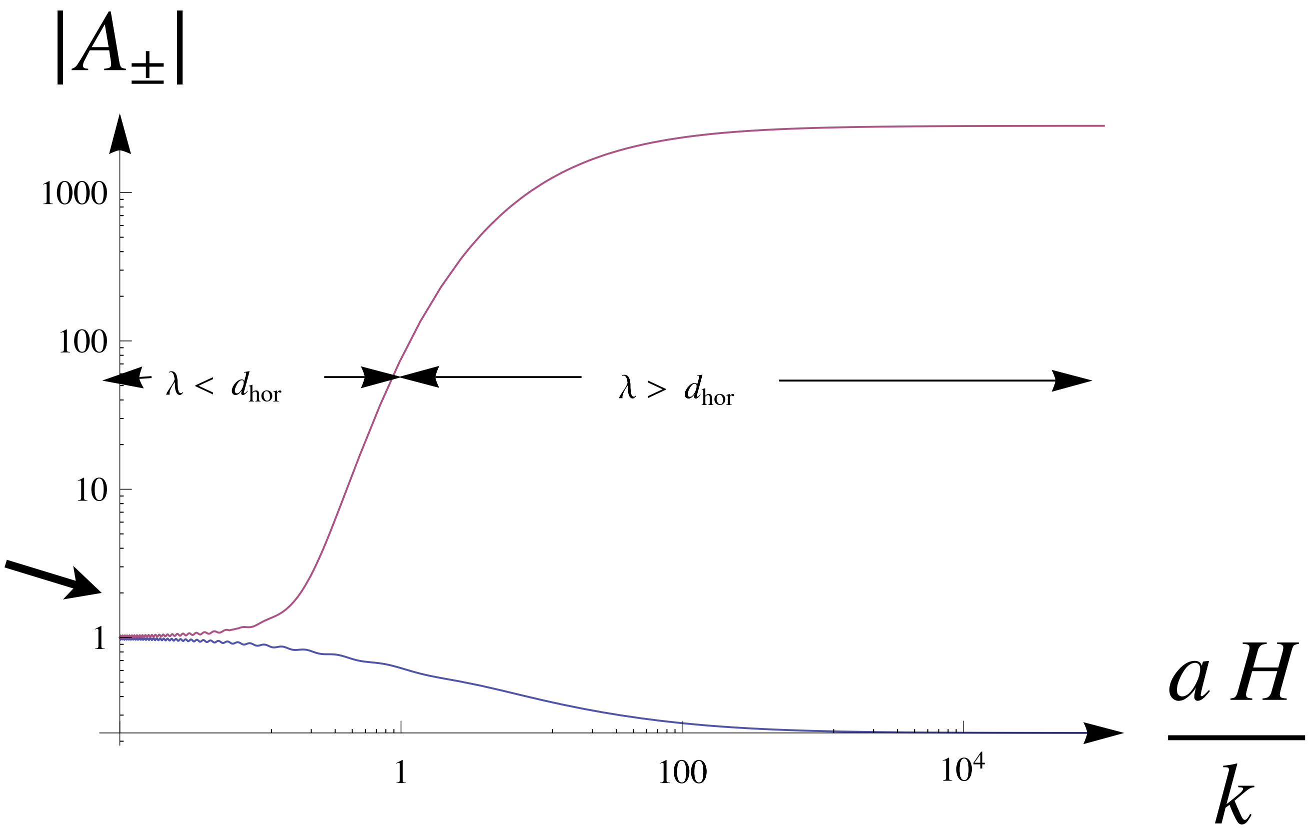
$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\phi^{(0)}\ F\,\tilde{F}$$

$$A''_{\pm} + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_{\pm} = 0$$

helicity

$a\,\dot{\phi}^{(0)}$

Vacuum

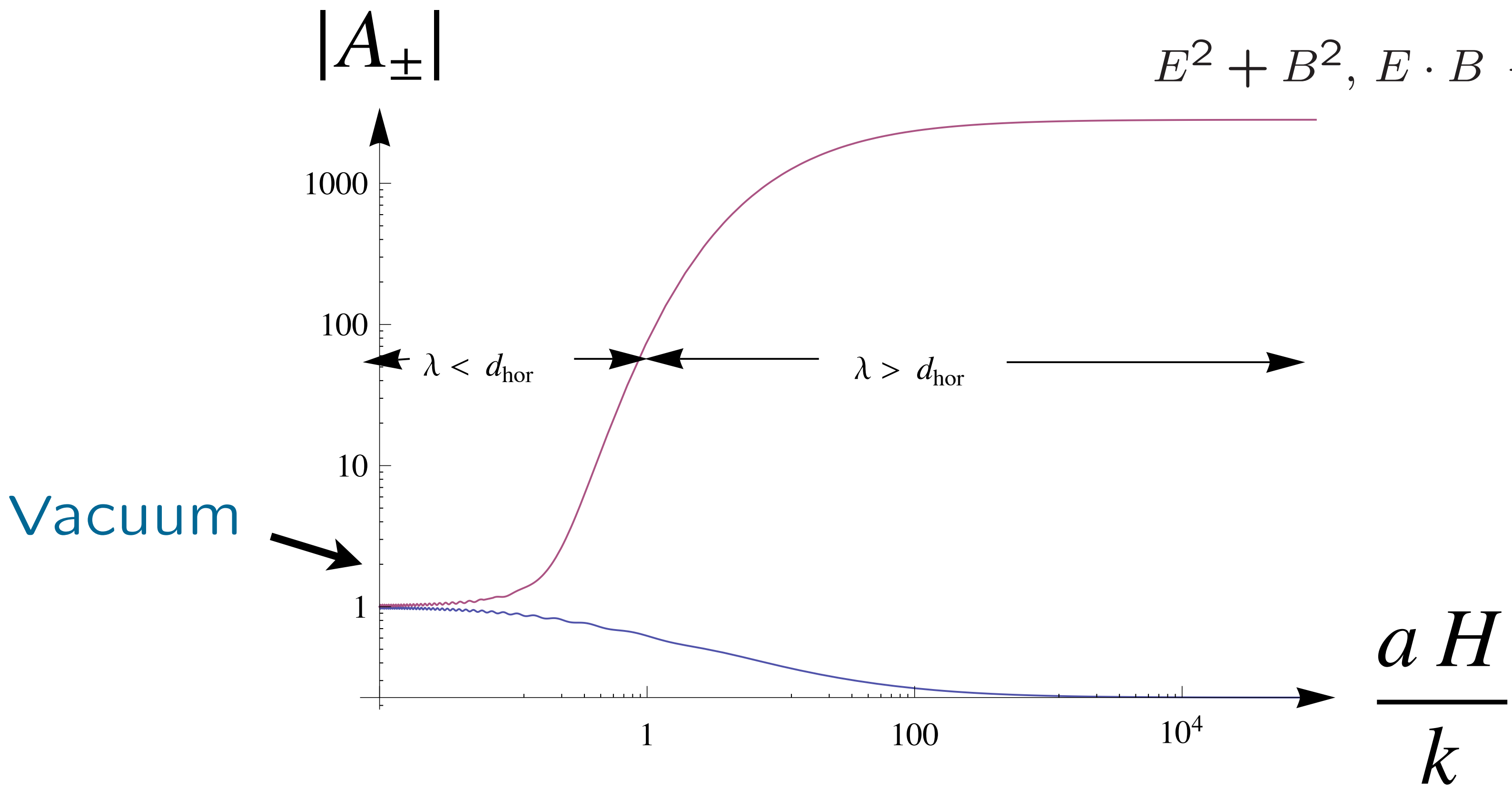


$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\phi^{(0)}\; F\,\tilde{F}$$

$$A''_\pm + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_\pm = 0$$

helicity

$a\,\dot{\phi}^{(0)}$

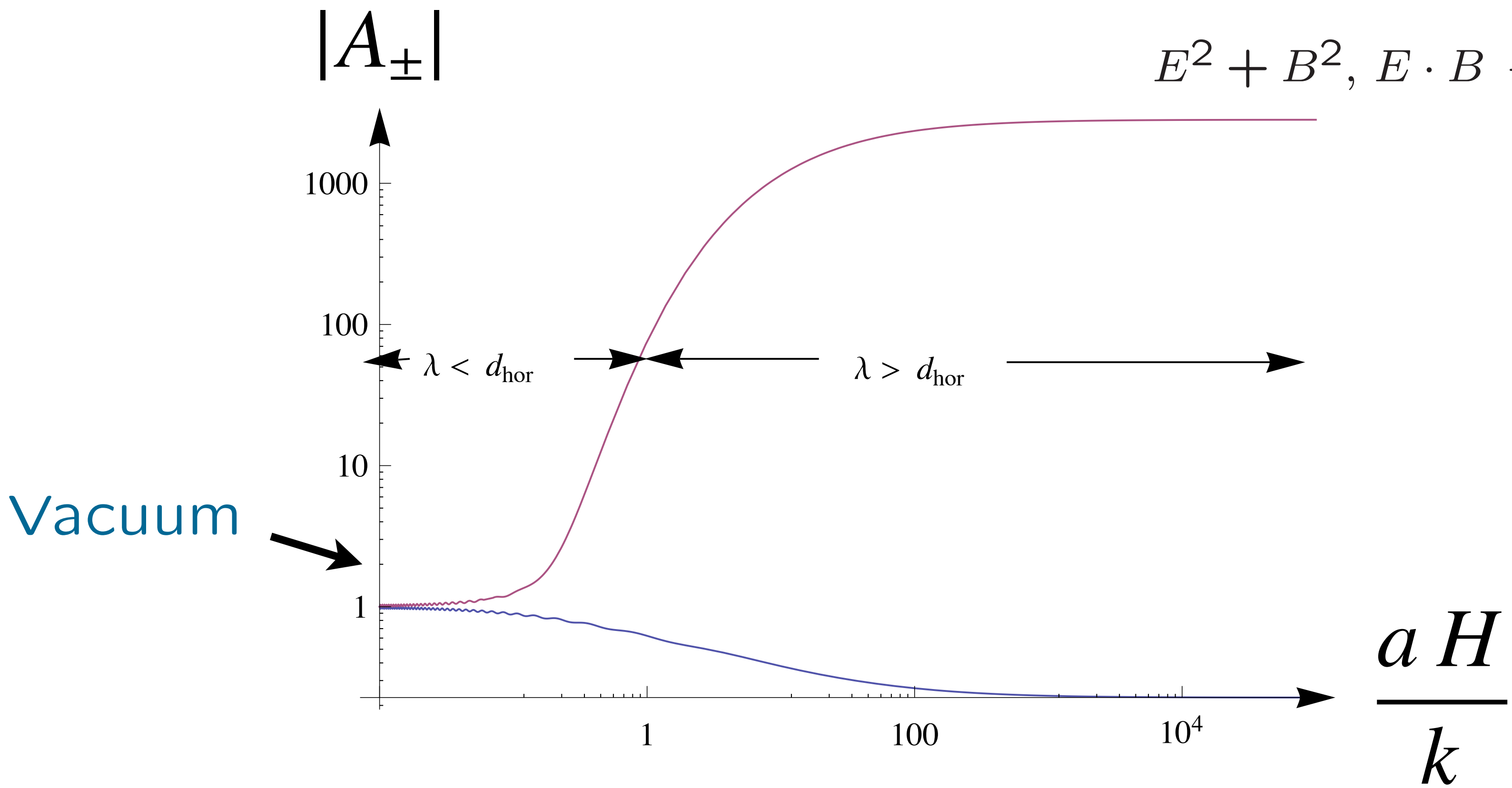


$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{\alpha}{f}\phi^{(0)}\; F\,\tilde{F}$$

$$A''_\pm + \left[k^2 \mp k\,\frac{\alpha}{f}\,\phi^{(0)\prime}\right] A_\pm = 0$$

helicity

$a\,\dot{\phi}^{(0)}$



Physical effects only from modes A_+ at horizon exit

- A production from ϕ kinetic energy. Resulting friction can be so strong as to facilitate ϕ slow roll. Anber, Sorbo '09
- We impose negligible backreaction of A on background dynamics

- A production from ϕ kinetic energy. Resulting friction can be so strong as to facilitate ϕ slow roll. Anber, Sorbo '09
- We impose negligible backreaction of A on background dynamics

---

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} - \frac{\vec{\nabla}^2}{a^2}\delta\phi + m^2\delta\phi = \frac{\alpha}{f}\,F^{\mu\nu}\,\tilde{F}_{\mu\nu}$$

$$\delta\phi = \delta\phi_{\text{vacuum}} + \delta\phi_{\text{inv.decay}}$$

- A production from ϕ kinetic energy. Resulting friction can be so strong as to facilitate ϕ slow roll. Anber, Sorbo '09
- We impose negligible backreaction of A on background dynamics

---

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} - \frac{\vec{\nabla}^2}{a^2}\,\delta\phi + m^2\delta\phi = \frac{\alpha}{f}\,F^{\mu\nu}\,\tilde{F}_{\mu\nu}$$

FT $\searrow$ J_k

$$\delta\phi = \delta\phi_{\text{vacuum}} + \delta\phi_{\text{inv.decay}}$$

$$\widehat{\delta\phi}_{\text{inv.decay}}\,(\eta) = \int d\eta'\,G_k\left(\eta,\,\eta'\right)\,\widehat{J_k}\left(\eta'\right)$$

$$G_k = i\,\theta\left(\eta - \eta'\right)\,\delta\phi_k\,(\eta)\,\delta\phi_k^*\left(\eta'\right) + \text{h. c.}$$

- A production from ϕ kinetic energy. Resulting friction can be so strong as to facilitate ϕ slow roll. Anber, Sorbo '09
- We impose negligible backreaction of A on background dynamics

---

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} - \frac{\vec{\nabla}^2}{a^2}\,\delta\phi + m^2\delta\phi = \frac{\alpha}{f}\,F^{\mu\nu}\,\tilde{F}_{\mu\nu}$$

$\searrow$ FT

J_k

$$\delta\phi = \delta\phi_{\text{vacuum}} + \delta\phi_{\text{inv.decay}}$$

$$\widehat{\delta\phi}_{\text{inv.decay}}\,(\eta) = \int d\eta'\,G_k\left(\eta,\,\eta'\right)\,\widehat{J_k}\left(\eta'\right)$$

$$G_k = i\,\theta\left(\eta - \eta'\right)\,\delta\phi_k\,(\eta)\,\delta\phi_k^*\left(\eta'\right) + \text{h. c.}$$

$\Rightarrow$ $\langle\delta\phi_{\text{vacuum}}\;\delta\phi_{\text{inv.decay}}\rangle = 0$

- A production from ϕ kinetic energy. Resulting friction can be so strong as to facilitate ϕ slow roll. Anber, Sorbo '09
- We impose negligible backreaction of A on background dynamics

---

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} - \frac{\vec{\nabla}^2}{a^2}\,\delta\phi + m^2\delta\phi = \frac{\alpha}{f}\,F^{\mu\nu}\,\tilde{F}_{\mu\nu}$$

FT $\searrow$ J_k

$$\delta\phi = \delta\phi_{\text{vacuum}} + \delta\phi_{\text{inv.decay}}$$

$$\widehat{\delta\phi}_{\text{inv.decay}}(\eta) = \int d\eta'\,G_k\left(\eta,\,\eta'\right)\,\widehat{J_k}\left(\eta'\right)$$

$$G_k = i\,\theta\left(\eta-\eta'\right)\,\delta\phi_k(\eta)\,\delta\phi_k^*\left(\eta'\right) + \text{h. c.}$$

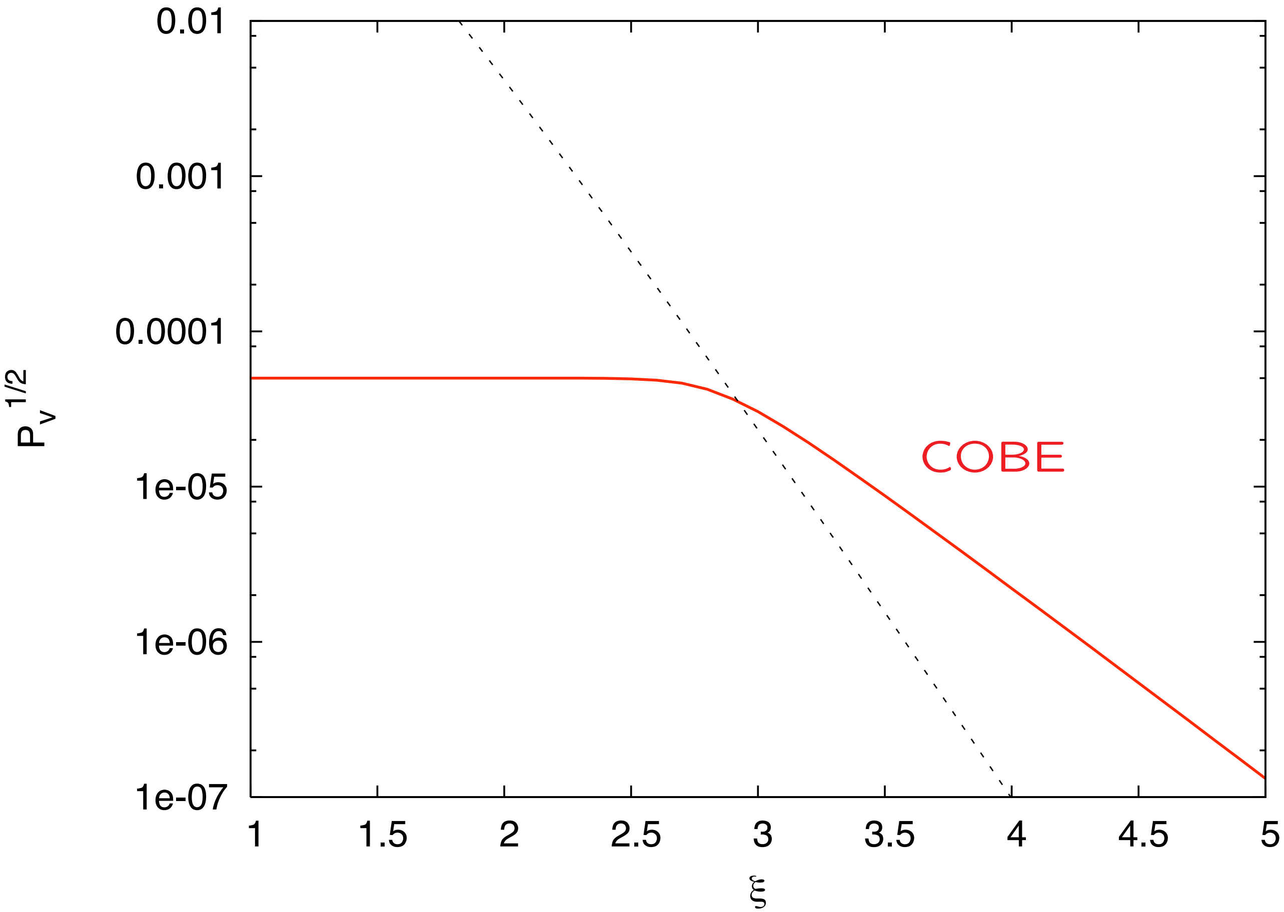
$\Rightarrow$ $\langle\delta\phi_{\text{vacuum}}\;\delta\phi_{\text{inv.decay}}\rangle = 0$

Same scale dependence

$$P_\zeta(k) = \mathcal{P}_v \left(\frac{k}{k_0}\right)^{n_s-1} \left[1 + 7.5 \cdot 10^{-5}\, \mathcal{P}_v \, \frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\mathcal{P}_v^{1/2} \equiv \frac{H}{2\pi|\dot{\phi}|}$$

$$\xi \equiv \frac{\alpha}{f}\frac{|\dot{\phi}|}{2H}$$

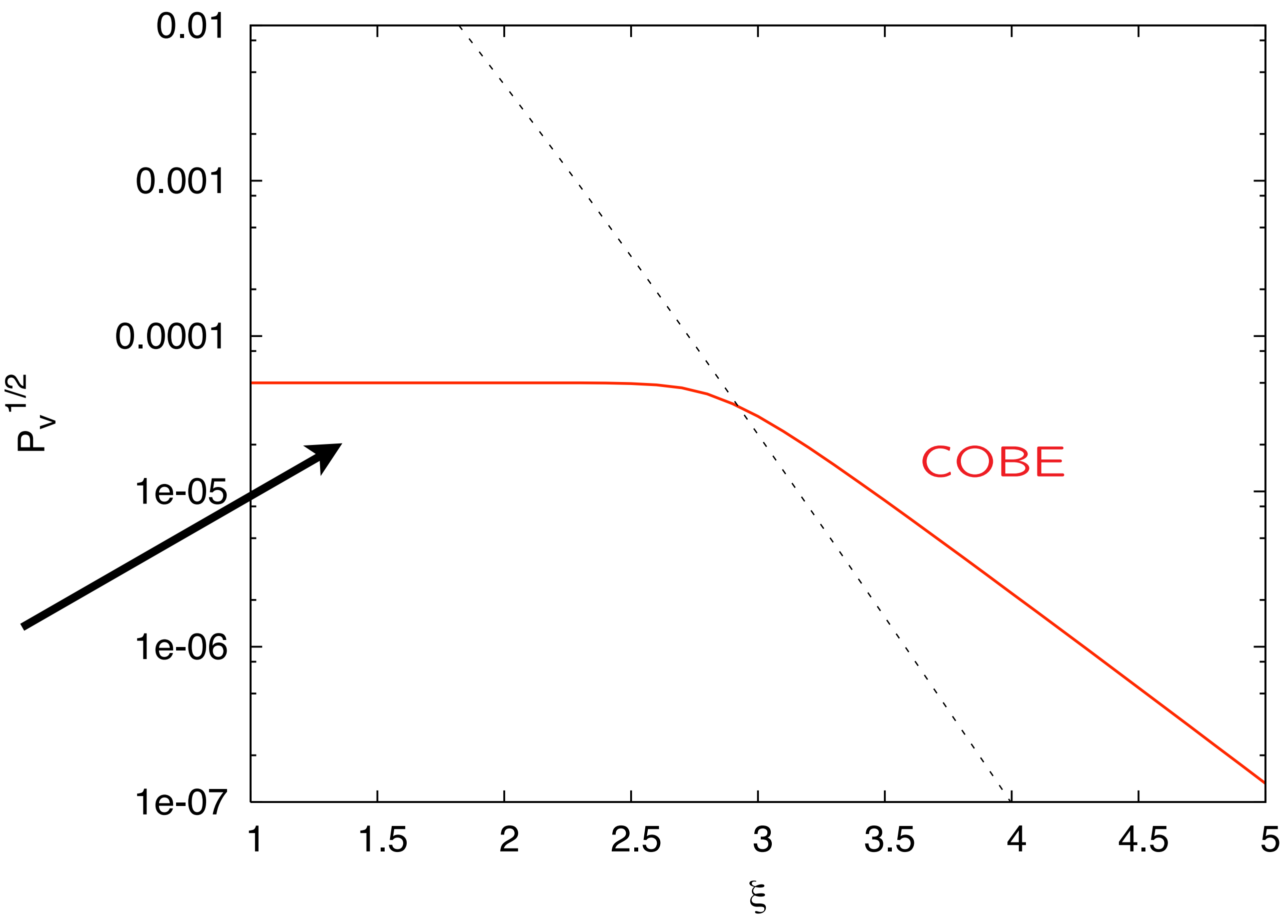


$$P_\zeta(k) = \mathcal{P}_v \left(\frac{k}{k_0}\right)^{n_s - 1} \left[1 + 7.5 \cdot 10^{-5}\, \mathcal{P}_v \frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\mathcal{P}_v^{1/2} \equiv \frac{H}{2\pi|\dot{\phi}|}$$

$$\xi \equiv \frac{\alpha}{f} \frac{|\dot{\phi}|}{2H}$$

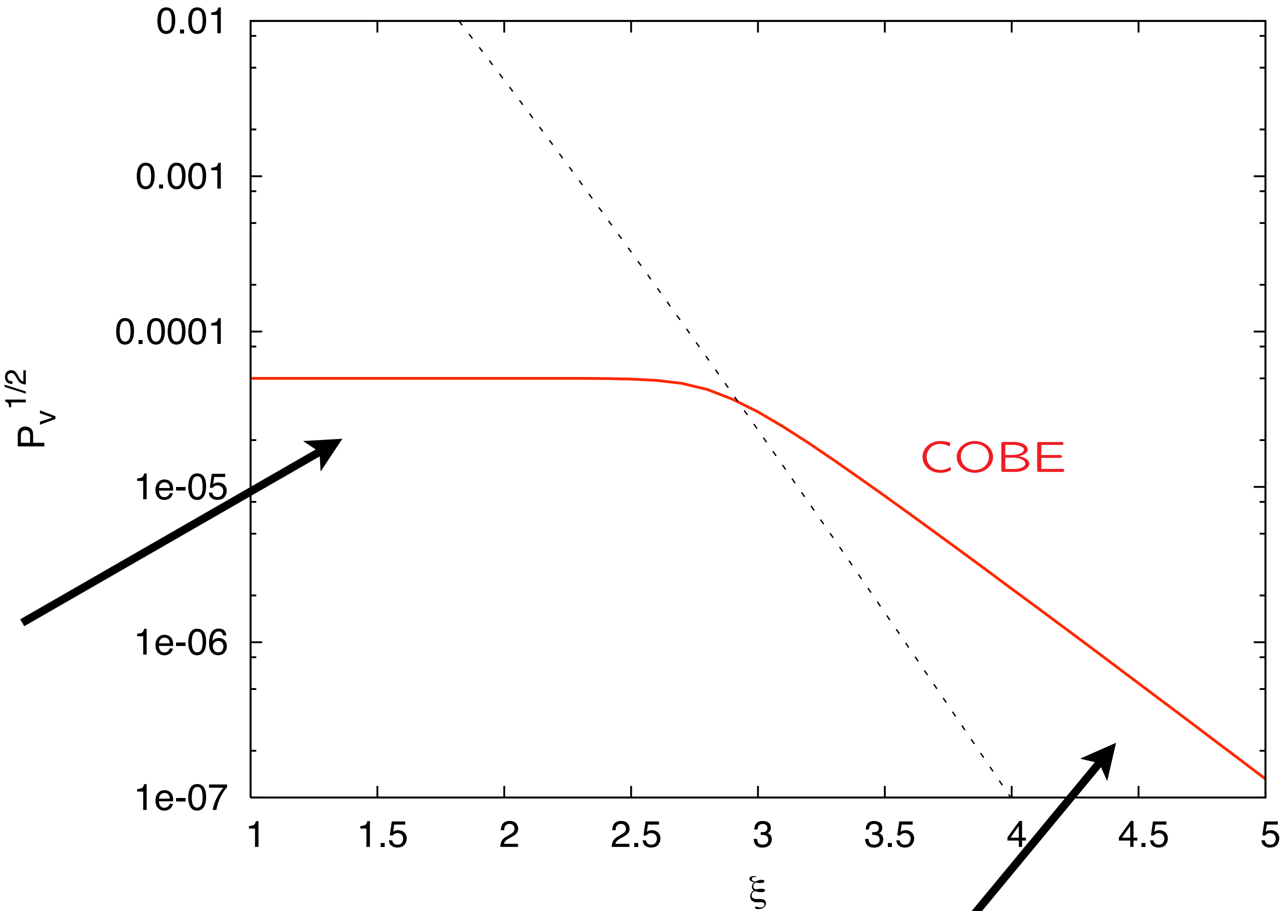
$\delta\phi_{\rm vac}$ dominates
standard result



$$P_\zeta(k) = \mathcal{P}_v \left(\frac{k}{k_0}\right)^{n_s-1} \left[1 + 7.5 \cdot 10^{-5}\, \mathcal{P}_v \frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\mathcal{P}_v^{1/2} \equiv \frac{H}{2\pi|\dot\phi|}$$

$$\xi \equiv \frac{\alpha}{f}\frac{|\dot\phi|}{2H}$$



$\delta\phi_{\rm vac}$ dominates
standard result

$\delta\phi_{\rm inv.decay}$ dominates
need to decrease $\mathcal{P}_v$ exponentially

Bispectrum $\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = B(k_i)\,\delta^{(3)}\left(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}\right)$

Scale invariance $\Rightarrow$ $B\,(k_i) = k_1^{-6}\,B\left(1, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right) \quad , \quad x_i \equiv \frac{k_i}{k_1}$

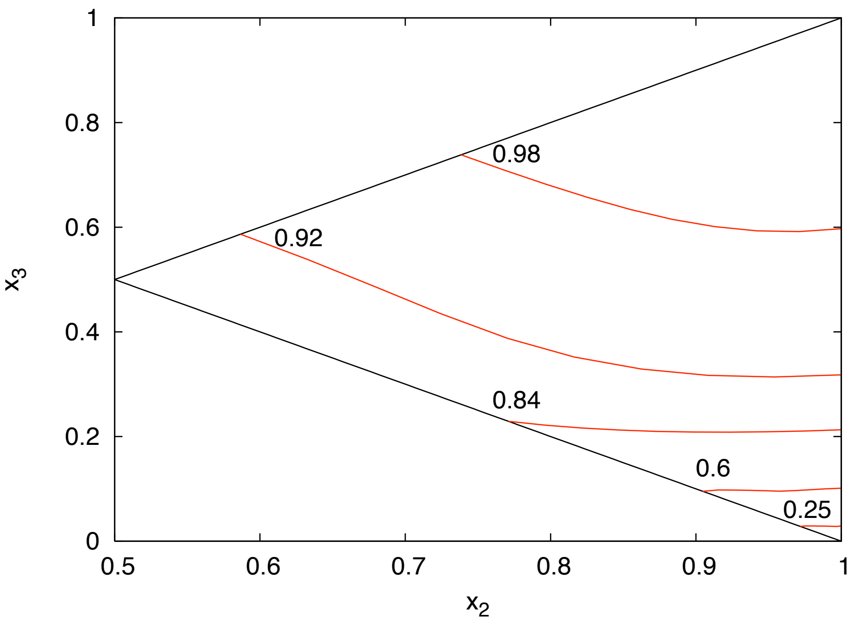
Shape $S\,(x_1, x_2) \propto B\,(1, x_2, x_3)\; x_2^2\, x_3^2$ Babich, Creminelli, Zaldarriaga '04

Bispectrum $\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = B(k_i)\, \delta^{(3)}\left(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}\right)$

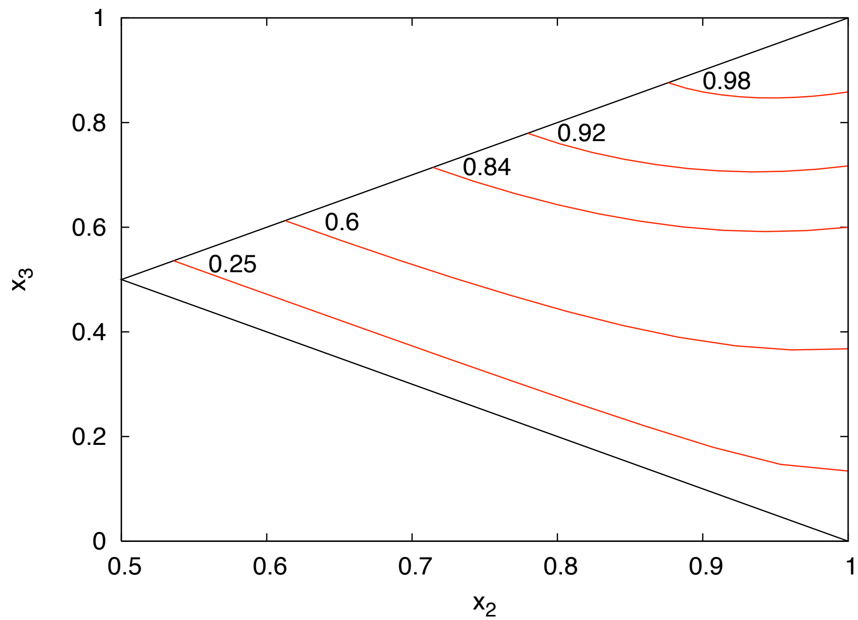
Scale invariance $\Rightarrow \quad B(k_i) = k_1^{-6}\, B\left(1, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right) \quad , \quad x_i \equiv \frac{k_i}{k_1}$

Shape $\quad S(x_1, x_2) \propto B(1, x_2, x_3)\, x_2^2\, x_3^2$ Babich, Creminelli, Zaldarriaga '04

Axion Inflation



Equilateral template



$x_3 \le x_2 \le 1$

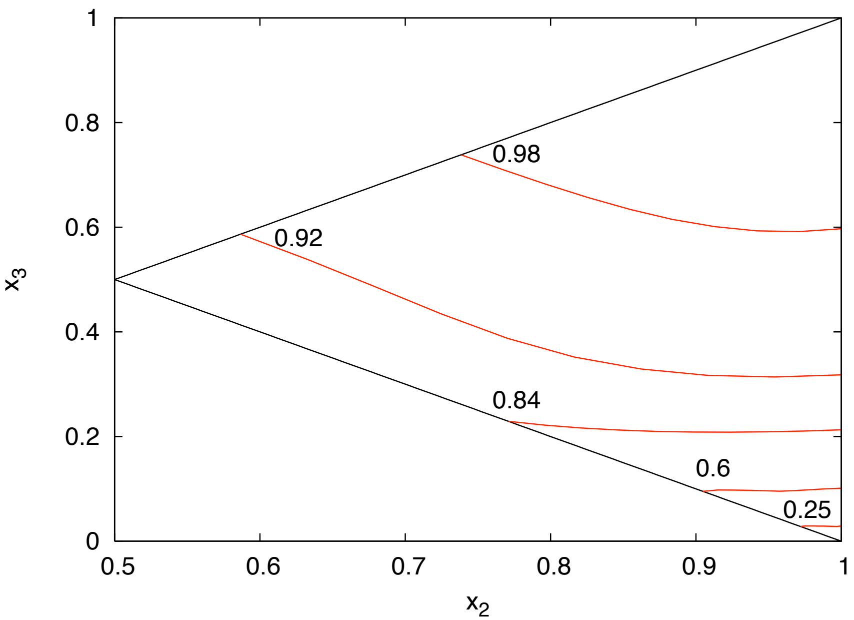
$x_2 + x_3 \ge 1$

Bispectrum $\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = B(k_i)\, \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$

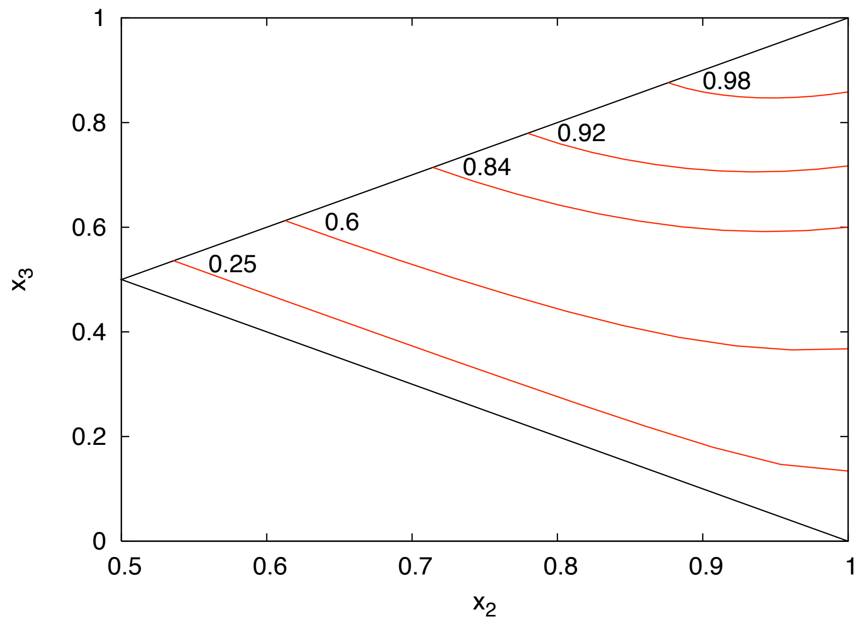
Scale invariance $\Rightarrow \quad B(k_i) = k_1^{-6}\, B\left(1, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right) \quad , \quad x_i \equiv \frac{k_i}{k_1}$

Shape $\quad S(x_1, x_2) \propto B(1, x_2, x_3)\, x_2^2\, x_3^2$ Babich, Creminelli, Zaldarriaga '04

Axion Inflation



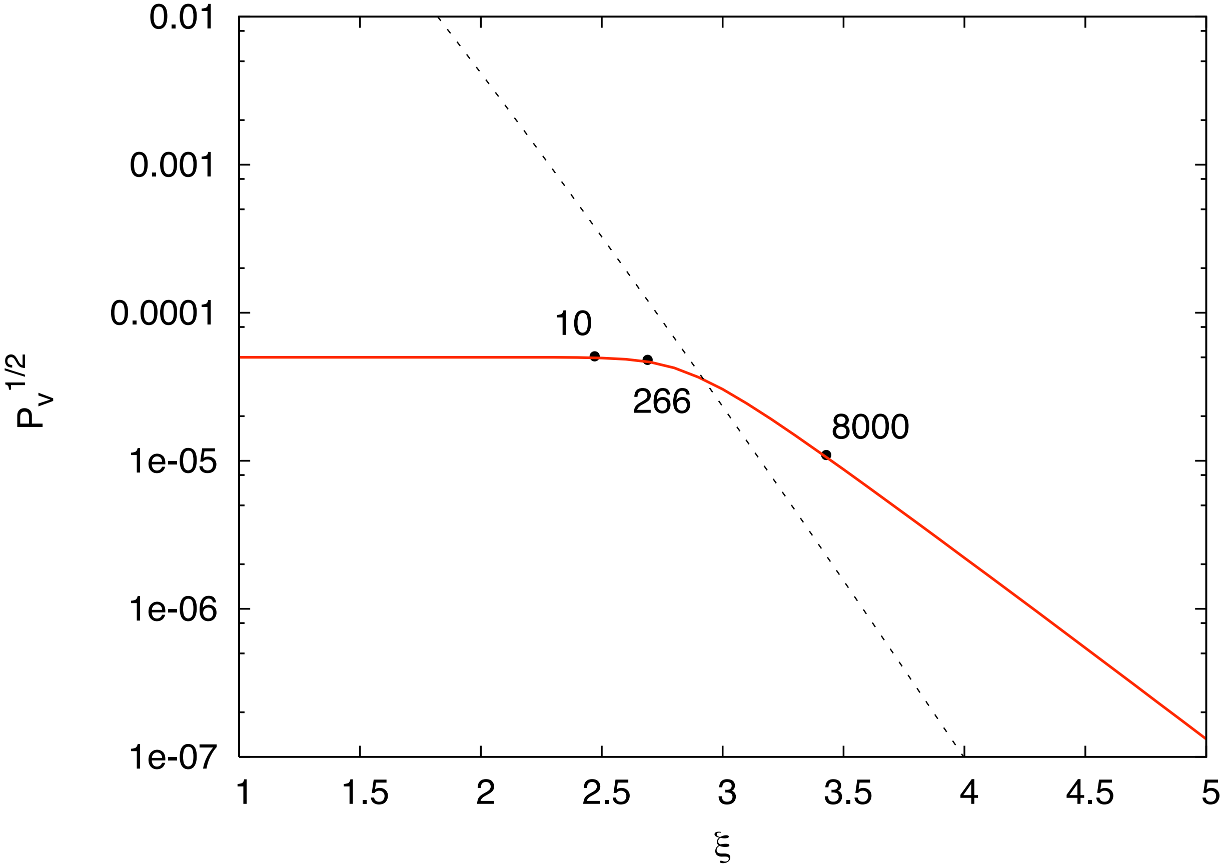
Equilateral template



$x_3 \le x_2 \le 1$

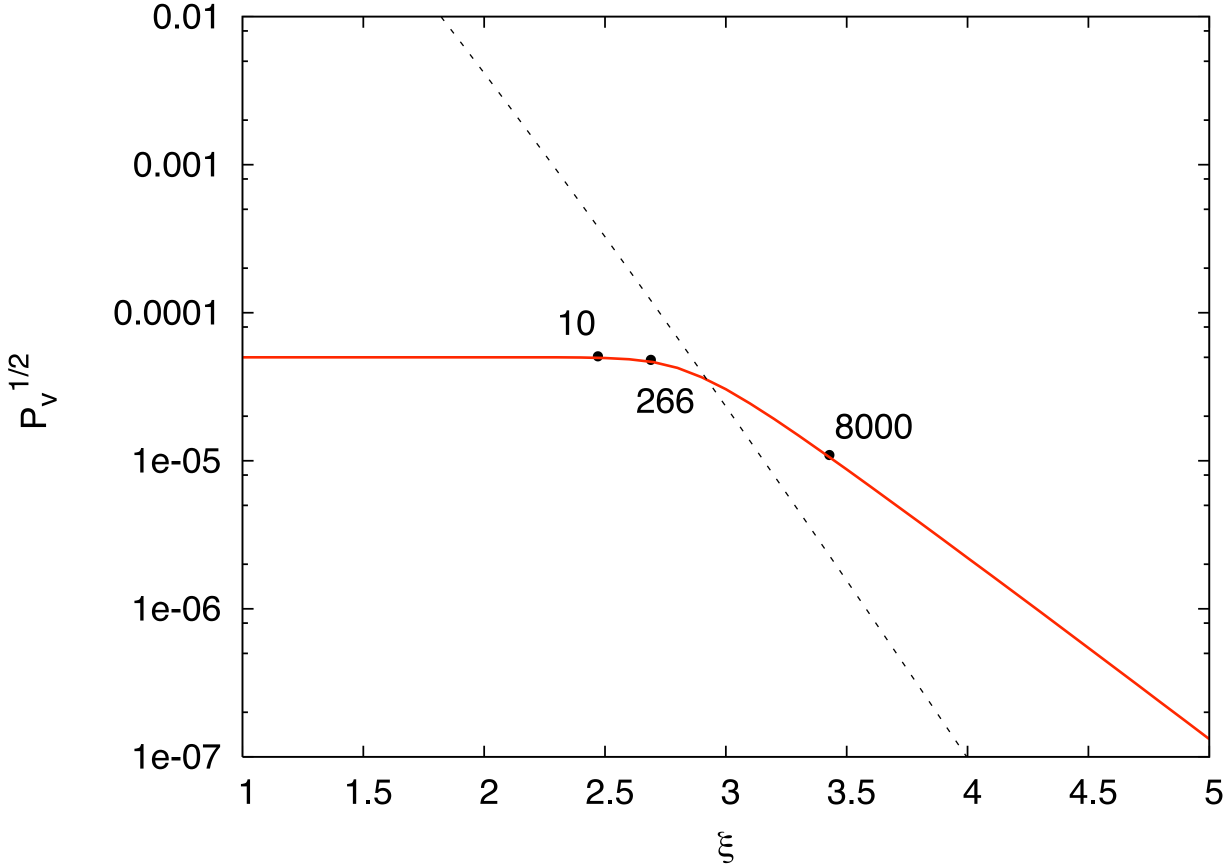
$x_2 + x_3 \ge 1$

cos (axion inf. , equil.) = 0.93 ; cos (axion inf. , orth.) = −0.15



$$f_{NL}^{\text{equil}} \simeq 4.4 \cdot 10^{10}\, \mathcal{P}_v^3 \frac{e^{6\pi\xi}}{\xi^9}$$

$$+ O(\epsilon, \eta)$$



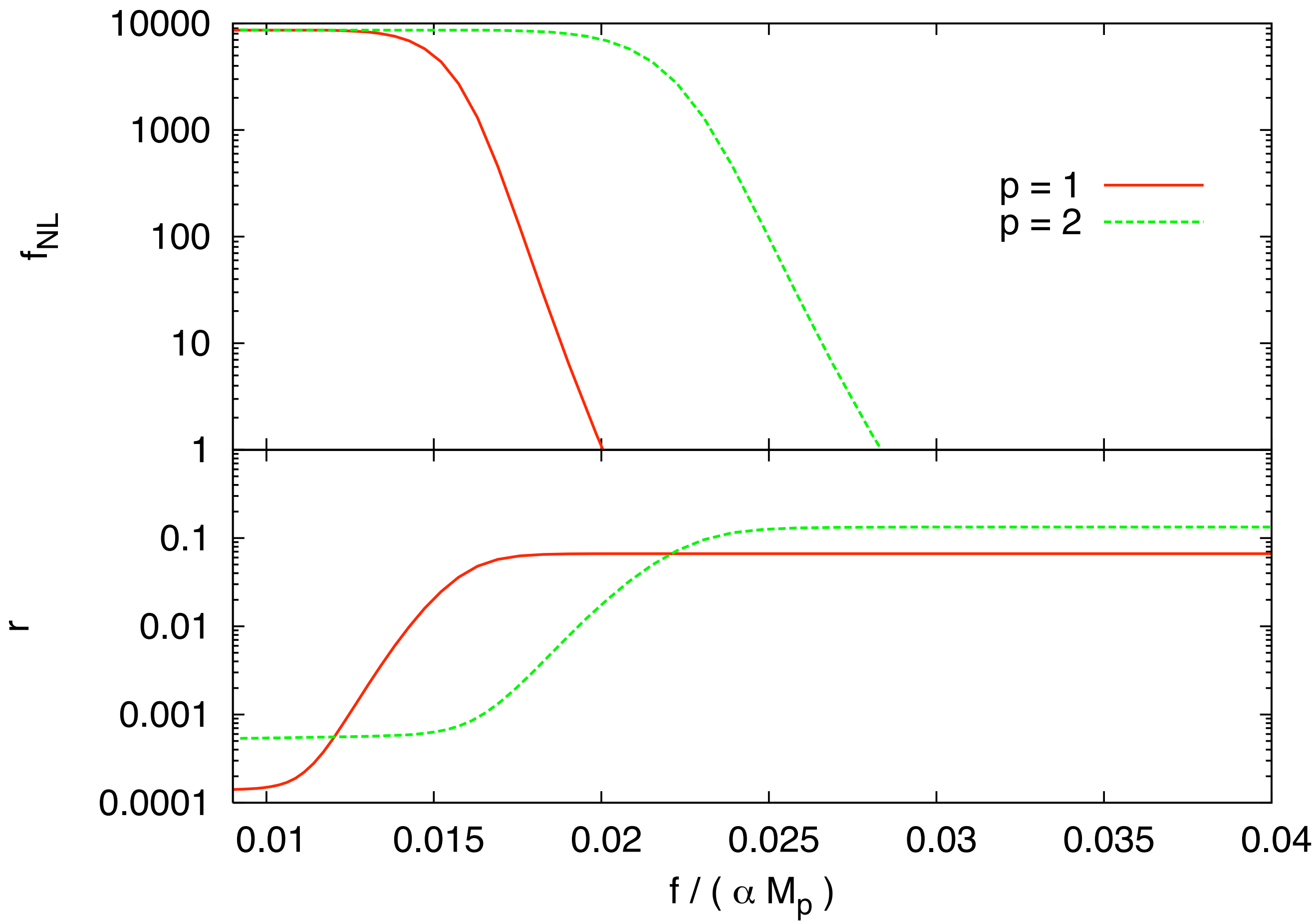
$$f_{NL}^{\text{equil}} \simeq 4.4 \cdot 10^{10} \, \mathcal{P}_v^3 \, \frac{e^{6\pi\xi}}{\xi^9} + O(\epsilon, \eta)$$

$$P_{\text{GW}} \simeq \frac{2H^2}{\pi^2 M_p^2} \left(\frac{k}{k_0} \right)^{n_T} \left[1 + 4.3 \cdot 10^{-7} \, \frac{H^2}{M_p^2} \, \frac{e^{4\pi\xi}}{\xi^6} \right]$$

Single helicity $\rightarrow \langle BE \rangle, \langle BT \rangle$ Sorbo '11 ; Kamionkowski, Souradeep '10

However, strongly subdominant in region allowed by f_{NL}

So far, H and $\dot{\phi}$ unrelated; now specify to $V \propto \phi^p$



Obervational Bounds:

- $-214 < f_{NL}^{\rm equil} < 266$
- $r \lesssim 0.2$

Detection of both r and f_{NL} for $f \sim 10^{-2}\,\alpha\,M_p$

Natural value in controlled realizations of axion inflation